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FOUNDATION LEVEL
PAPER 3: BUSINESS MATHEMATICS, LOGICAL REASONING & STATISTICS

Matrices

Faculty: CA Arijit Chakraborty

INTRODUCTION TO MATRIX ALGEBRA

DEFINITION OF A MATRIX

A matrix is a rectangular array of quantities arranged in rows and columns. A matrix containing *m* rows and *n* columns can be expressed as

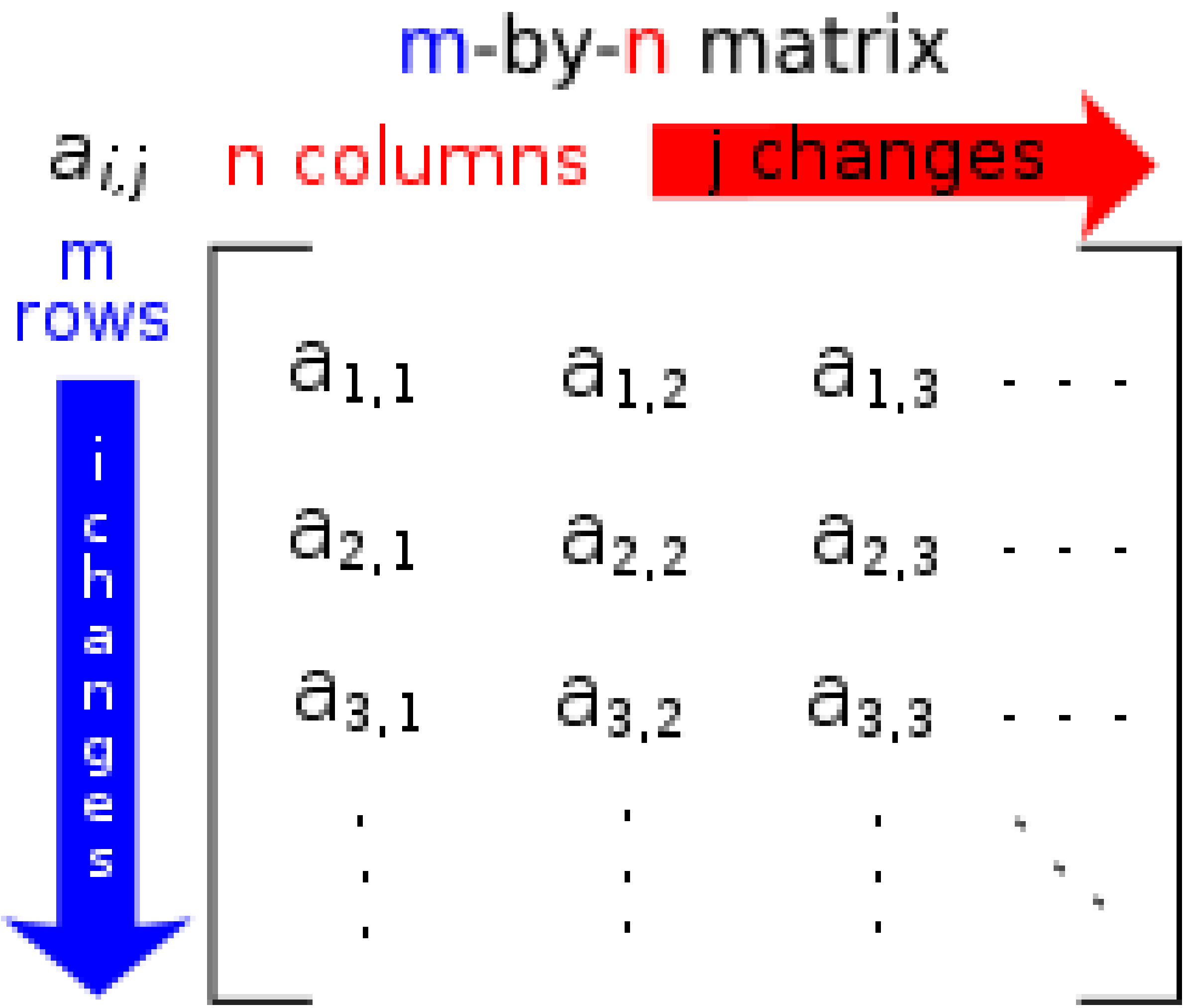
$$\mathbf{A} = [A] = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1n} \\ A_{21} & A_{22} & \cdots & A_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ A_{m1} & A_{m2} & \cdots & A_{mn} \end{bmatrix}_{mxn}$$

The quantities that form a matrix are referred to as elements of the matrix. Each element of the matrix is identified with two subscripts i and j to designate the row and column location, respectively. Thus, the i,j element (or coefficient) of [A] is expressed as A_{ij}

Concept

- A is rectangular matrix with m rows and n columns. The numbers aij, i = 1,2 ……..m; j = 1,2,…..n of this array are called its elements aij, is associated.
- We shall denote a matrix either using by using brackets
- [ ]; or ( ).

$$
\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ \vdots \\ m \end{array}
\begin{array}{c}
\begin{array}{cccc} 1 & 2 & \cdots & n \end{array} \\
\begin{bmatrix}
a_{11} & a_{12} & \cdots & a_{1n} \\
a_{21} & a_{22} & \cdots & a_{2n} \\
a_{31} & a_{32} & \cdots & a_{3n} \\
\vdots & \vdots & \vdots & \vdots \\
a_{m1} & a_{m2} & \cdots & a_{mn}
\end{bmatrix}
\end{array}
$$


m-by-n matrix
$a_{i,j}$
n columns
j changes
m rows
i changes
$a_{1,1}$ $a_{1,2}$ $a_{1,3}$ . . .
$a_{2,1}$ $a_{2,2}$ $a_{2,3}$ . . .
$a_{3,1}$ $a_{3,2}$ $a_{3,3}$ . . .

Matrix Method

- Matrices applications are used in Business, Finance and Economics. Matrices applications are helpful to solve the linear equations with the help of this cost estimation, sales projection, etc., can be predicted.

- **Matrix: pg 2.36 , study material**
- Ram, Sita and Laxman are three friends. Ram has 5 books, 3 pencils and 2 pens. Sita has 10 books, 8 pencils and 5 pens. Laxman has 15 books, 10pencils and 2 pens. The above information about three friends can be represented in the following form:

Matrix Formation

	Books	Pencils	Pen
Ram	5	3	2
Sita	10	8	5
Laxman	15	10	2

Note:

- It is to be noted that a matrix is **just an arrangement of elements** without any value in rows and columns.
- The **plural matrix is matrices.**
- **Order of a Matrix:** A matrix A with m rows and n columns is called a matrix of order (m, n) or $m \times n$ (read as m by n).

TYPES OF MATRICES

- **Row Matrix:**
- A matrix which has only one row is called a row matrix or row vector.
- The matrices of the type [a1, a2, a3 ,an]; [1, 2, 5] are examples of row matrices.
- **Column Matrix:**
- A matrix which has only one column is called a column matrix or a column vector.
- **Zero matrix / Null Matrix**
- If every element of a m × n matrix is zero, the matrix is called zero matrix or null matrix of order

Types of Matrices & illustration pg 2.36-37

- **Square Matrix and Rectangular Matrix:** If the number of rows and columns in a matrix are same, such matrix is called a square matrix; otherwise it is called a rectangular matrix
- Square matrix – 4 by 4, 3by 3 , 2 by 2
- **Scalar Matrix:**
- A diagonal matrix whose leading diagonal elements are all equal is called a scalar matrix - **let us create scalar matrix**
- **(k00, 0k0, 00k)**

Unit Matrix – pg 2.37

- **Unit Matrix:**
- A scalar matrix whose **diagonal elements are equal to unity** is called unit matrix and it is denoted by $I_{n \times n}$, if it is order of order n
- Let us create
- (1000
- 0100
- 0010
- 0001)

More types pg 2.38

- **Upper triangle matrix:**
- A matrix is known as upper triangular matrix if all the elements **below** the leading diagonal are zero
- Let us create
- (165, 034, 002)
- **Lower Triangular Matrix:**
- A matrix is known as lower triangular matrix if all the elements **above the leading diagonal are zero**
- Let us create
- **(100, 670, 798)**

Sub Matrix: pg 2.38

- The matrix obtained by deleting one or more rows or columns or both of a matrix is called its sub matrix.
- Lets create sub-matrix
- From (767 320 541)
- Sub matrix = (7 7 51)

Equal Matrices: pg 2.38

- Two matrices A=[aij] and B=[bij] are said to be equal if they satisfy the following two conditions.
- 1. The order of both the matrices is same;
- 2. Corresponding elements in both the matrices are equal

Symmetric matrix

- In linear algebra, a **symmetric matrix** is a square matrix that is equal to its transpose. Formally,
- If $A = A^T$
- Eg A= (1 7 3, 7 4 –5, 3 --5 6)

Skew-symmetric matrix

- A **skew-Symmetric** (or **antisymmetric** or **antimetric** **matrix** is a square matrix whose transpose equals its negative
- $A^T = -A$
- A ji = - A ij
- A = (0 2 --45, --2 0 --4,45 4 0)

Principal Diagonal of a matrix

- The straight path from first element of first row to last element of last row of a square matrix is called **principal diagonal** of the matrix.
- The **principal diagonal of a square matrix is also called as leading diagonal of the matrix.**
- A straight path can be formed by joining the two elements which are at two opposite corners of a square matrix.
- One corner is position of **first element of first row and another corner is position of last element of last row.**
- The diagonal which joins them contains some elements **in the case of square matrices** which are **higher than second order square matrix.**

Unit Matrix – pg 2.37

- **Unit Matrix: Identity Matrix**
- A scalar matrix whose **diagonal elements are equal to unity** is called unit matrix and it is denoted by In×n, if it is order of order n
- Let us create
- (1000 or (100 , 010, 001) =order 3
- 0100 or (10, 01) = order 2
- 0010
- 0001)
- Order 4

Determinant – Ex 3

The **determinant of a** **<u>square matrix</u> A** is the integer obtained through a range of methods using the elements of the matrix.
It is a no. associated with the square matrix

$$\det\begin{bmatrix} -5 & -4 \\ -2 & -3 \end{bmatrix} = (-5)(-3)-(-4)(-2)$$
$$= 15 - 8$$
$$= 7 ✓$$

ALGEBRA OF MATRICES (unit 2.2.3,pg 2.38, 39)

- **Addition and Subtraction of matrices:**
- Let A and B be two matrices of the same order. Then the addition of A and B, denoted by A+B, is the matrix obtained by adding corresponding entries of A and similarly to subtract two matrices we just subtract their corresponding elements
- **Remark:** We can add two matrices of the same order. If they are of the same order, we say they are comfortable for addition. Also, the order of the matrices is the same as that of the two original matrices.

Matrix addition

- A= (1 3 -2, 4 2 3 , -1 5 4)
- B= (5 2 –1, 3 1 2 , 4 3 --2)
- C= A+B
- = (6 5 –3, 7 3 5 , 3 8 2)

Matrix Addition & Subtraction

- Only possible for matrices of same dimension
- Add/subtract matrices element-by-element
- Addition example: **C = A+B**

$$\begin{bmatrix} 1 & 3 & -2 \\ 4 & 2 & 3 \\ -1 & 5 & 4 \end{bmatrix} + \begin{bmatrix} 5 & 2 & -1 \\ 3 & 1 & 2 \\ 4 & 3 & -2 \end{bmatrix} = \begin{bmatrix} 6 & 5 & -3 \\ 7 & 3 & 5 \\ 3 & 8 & 2 \end{bmatrix}$$

- Subtraction example: **C = A-B**

$$\begin{bmatrix} 4 & 2 & -1 \\ 5 & 3 & -3 \end{bmatrix} - \begin{bmatrix} 1 & 4 & 2 \\ -4 & 3 & 3 \end{bmatrix} = \begin{bmatrix} 3 & -2 & -3 \\ 9 & 0 & -6 \end{bmatrix}$$

Property

- If A, B, C are matrices of same order, then
  - $A + B = B + A$ *(Commutative Law)*
  - $(A + B) + C = A + (B + C)$ *(Associative Law)*
  - $K(A + B) = K.A + K.B$, *where K is constant*

Matrix Algebra Rules

A Lot Like Normal Algebra Rules

$A + B = B + A$	(Addition Commutes)
$A + (B + C) = (A + B) + C$	(Addition Associates)
$A(BC) = (AB)C$	(Multiplication Associates)
$A(B + C) = AB + AC$	(Distribute)
$c(A + B) = cA + cB$	(Scalars Distribute)
$(c + d)A = cA + dA$	

Notice:

$AB \neq BA$ (Multiplication **DOES NOT** Commute)

Example 8

Omega Tyre store has two store locations and and their sales of tyres are given by make (in rows) and quarters (in columns) as shown below.

$$[A] = \begin{bmatrix} 25 & 20 & 3 & 2 \\ 5 & 10 & 15 & 25 \\ 6 & 16 & 7 & 27 \end{bmatrix} \qquad [B] = \begin{bmatrix} 20 & 5 & 4 & 0 \\ 3 & 6 & 15 & 21 \\ 4 & 1 & 7 & 20 \end{bmatrix}$$

where the rows represent the sale of A, B and C tires respectively and the columns represent the quarter number: 1, 2, 3 and 4. What are the total tire sales for the two locations by make and quarter?

Example 8 (cont.)

$$[C]=[A]+[B]$$

$$=\begin{bmatrix} 25 & 20 & 3 & 2 \\ 5 & 10 & 15 & 25 \\ 6 & 16 & 7 & 27 \end{bmatrix}+\begin{bmatrix} 20 & 5 & 4 & 0 \\ 3 & 6 & 15 & 21 \\ 4 & 1 & 7 & 20 \end{bmatrix}$$

$$=\begin{bmatrix} (25+20) & (20+5) & (3+4) & (2+0) \\ (5+3) & (10+6) & (15+15) & (25+21) \\ (6+4) & (16+1) & (7+7) & (27+20) \end{bmatrix}$$

Example 8 (cont.)

The answer then is,

$$= \begin{bmatrix} 45 & 25 & 7 & 2 \\ 8 & 16 & 30 & 46 \\ 10 & 17 & 14 & 47 \end{bmatrix}$$

So if one wants to know the total number of C tires sold in quarter 4 at the two locations, we would look at Row 3 – Column 4 to give $c_{34} = 47.$

Matrix Subtraction

Two matrices $[A]$ and $[B]$

$$[D]=[A]-[B]$$

Where

$$d_{ij}=a_{ij}-b_{ij}$$

Example 9- Subtraction

Subtract matrix $[B]$ from matrix $[A]$.

$$[A] = \begin{bmatrix} 5 & 2 & 3 \\ 1 & 2 & 7 \end{bmatrix}$$

$$[B] = \begin{bmatrix} 6 & 7 & -2 \\ 3 & 5 & 19 \end{bmatrix}$$

Example 9 (cont.)

$$[D] = [A] - [B]$$

$$= \begin{bmatrix} 5 & 2 & 3 \\ 1 & 2 & 7 \end{bmatrix} - \begin{bmatrix} 6 & 7 & -2 \\ 3 & 5 & 19 \end{bmatrix}$$

$$= \begin{bmatrix} (5-6) & (2-7) & (3-(-2)) \\ (1-3) & (2-5) & (7-19) \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -5 & 5 \\ -2 & -3 & -12 \end{bmatrix}$$

Matrix Multiplication

Two matrices $[A]$ and $[B]$ can be multiplied only if the number of columns of $[A]$ is equal to the number of rows of $[B]$ to give

$$[C]_{m\times n} = [A]_{m\times p}[B]_{p\times n}$$

If $[A]$ is a $m\times p$ matrix and $[B]$ is a $p\times n$ matrix, the resulting matrix $[C]$ is a $m\times n$ matrix.

Example 11 – multiplication

Given the following two matrices,

$$[A]=\begin{bmatrix}5 & 2 & 3\\ 1 & 2 & 7\end{bmatrix} \qquad [B]=\begin{bmatrix}3 & -2\\ 5 & -8\\ 9 & -10\end{bmatrix}$$

Find their product,

$$[C]=[A][B]$$

Example 11 (cont.)

c_{12} be found by multiplying the first row of $[A]$ by the second column of , $[B]$

$$c_{12} = \begin{bmatrix} 5 & 2 & 3 \end{bmatrix} \begin{bmatrix} -2 \\ -8 \\ -10 \end{bmatrix}$$

$$= (5)(-2) + (2)(-8) + (3)(-10)$$

$$= -56$$

Example 11 (cont.)

Similarly, one can find the other elements of $[C]$ to give

$$[C]=\begin{bmatrix}52 & -56\\ 76 & -88\end{bmatrix}$$

Multiplication of two matrices.

- The product A B of two matrices A and B defined only if the number of columns in Matrix A is equal to the number of rows in Matrix B.
- **Properties of matrix Multiplication:**
- Matrix multiplication is not commutative in general, i.e. $AB \neq BA$.
- Matrix multiplication is associative $(AB)\,C = A(BC)$, where both sides are defined.
- Multiplication distributes over addition of Matrices i.e.,
  - $A\,(B + C) = AB + AC$
  - $(A + B)\,C = AC + BC$

Properties

- If A, B and C are three matrices such that AB = AC , then the general B ≠ C.
- If A is m×n matrix and O is an n × p null matrix, then AO = O, A= O
- If A is a square matrix and I is a unit matrix of the same order, then AI = IA = O
- Product of the two no-zero matrices is non zero matrix.

Is [A][B]=[B][A]?

If [A][B] exists, number of columns of [A] has to be same as the number of rows of [B] and if [B][A] exists, number of columns of [B] has to be same as the number of rows of [A].

Now for [A][B]=[B][A], the resulting matrix from [A][B] and [B][A] has to be of the same size. This is only possible if [A] and [B] are square and are of the same size. Even then in general [A][B]≠[B][A].

Example 13

Determine if

$$[A][B]=[B][A]$$

for the following matrices

$$[A]=\begin{bmatrix} 6 & 3 \\ 2 & 5 \end{bmatrix}, \quad [B]=\begin{bmatrix} -3 & 2 \\ 1 & 5 \end{bmatrix}$$

Example 13 (cont.)

$$[A][B] = \begin{bmatrix} 6 & 3 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} -3 & 2 \\ 1 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} -15 & 27 \\ -1 & 29 \end{bmatrix}$$

$$[B][A] = \begin{bmatrix} -3 & 2 \\ 1 & 5 \end{bmatrix} \begin{bmatrix} 6 & 3 \\ 2 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} -14 & 1 \\ 16 & 28 \end{bmatrix}$$

Therefore

$$[A][B] \neq [B][A]$$

Pg 2.41, Eg 6

- A= (1 0 0, 0 1 0, A B –1)
- A^2 = ?
- A^2= A.A
- Work out =

- (1 0 0, 0 1 0, 0 0 1)
- = Identity matrix

Properties of transpose of a Matrix:

- A matrix is transpose of its matrix i.e. A = (A')'.
- The transpose of the sum of the two matrices is the sum of their transpose matrices, i.e. (A + B)'= A' + B'
- Transpose of a multiplication of a matrix and constant number is equal to the multiplication of the constant number by the transpose of matrix, i.e. (KA)' = K.A'
- The transpose of the two matrices are equal to the product of their transpose in reverse order, i.e., (AB)' = B'. A'

DETERMINANTS
(Pg 2.46 of Study Material)

- The determinant of a square matrix is a number which is associated with the square matrix. This number may be positive, negative or zero > the determinant of a square matrix A commonly denoted by :
- det A or
- |A| or
- Δ.
- The matrices which are not square do not have determinants.
- Determinants are quite useful to solving a system of linear equations. They are also equations. They are also helpful in expressing certain formulas.

- Let us take 2* 2 Matrix = ab cd
- Determinant = (ad-bc)

- Determinant of 3*3 matrix = Example : Boardwork

Minor of Matrix 3 by 3

- The **determinant** obtained through the **elimination of some rows and columns in a <u>square</u> matrix** is called a minor of that matrix.
- **Minor of the element of a determinant is the determinant of Mij by deleting ith row and jth column in which <u>element is existing</u>.**
- Matrix M= (1 4 2 , 5 3 7,6 2 1)
- One of the minors of the matrix = **det** (1 4, 5 3)
- (obtained through the elimination of row 3 and column 3 from the matrix
- Another minor is det (1 2 , 6 1) = obtained through the elimination of row 2 and column 2 from the matrix

Minor of **Matrix 4 by 4**

- Let P = (2 5 1 3, 4 1 7 9, 6 8 3 2 , 7 8 1 4)
- One of the minors of the matrix = obtained through the elimination of row 1 and column 1 from the matrix
- **Det** (1 7 9, 8 3 2 , 8 1 4)
- Another minor is : **det** (1 7, 8 3) : obtained through the elimination of **rows 1 and 4** and **columns 1 and 4 from the matrix**

Properties of Determinants: pg 2.47 (Study material

- The value of determinant **remains unaltered** interchanged if its rows or columns interchanged.
- The value of determinant **change signs** if any two rows (or columns) interchanges.
- The value of determinant is zero if any two rows (any columns) then value of determinant is equal to zero.
- The value determinant becomes **k times (where k is constant**) if any row or columns multiplied by k the value of determinant also multiplied by k.
- The value of determinant is zero if any two rows (or column) are proportional then the value of determinant is equal to zero.
- If each element of any row (or column) is **a sum of two numbers, the determinant can be expressed as the sum of the determinants.**
- The value of determinant remains same if to any (or column) multiple of row (or column) is added or subtracted.

1. Cramer's Rule - two equations

If we are given a pair of simultaneous equations

$$\begin{aligned} a_1x + b_1y &= d_1 \\ a_2x + b_2y &= d_2 \end{aligned}$$

then x, and y can be found from

$$x = \frac{\begin{vmatrix} d_1 & b_1 \\ d_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} \qquad y = \frac{\begin{vmatrix} a_1 & d_1 \\ a_2 & d_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

Short cut

$$x = \frac{\begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}}, \text{ and } z = \frac{\begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}}.$$

SOLUTION OF LINEAR EQUATIONS IN THREE VARIABLES (CRAMER'S RULE)

- The solution of equations
- $a_{11}\,x + a_{12}\,y + a_{13}\,z = d_1$

- $a_{21}x + a_{22}y + a_{23}z = d_2$
- $a_{31}x + a_{32}y + a_{33}z = d_3$

- is given by:
- x= DeT x / Det
- y= DeT y / Det
- Z= DeT z/ Det

Cramer's rule-example 14

Example

Solve the equations

$$\begin{aligned} 3x + 4y &= -14 \\ -2x - 3y &= 11 \end{aligned}$$

Solution 14

Solution

Using Cramer's rule we can write the solution as the ratio of two determinants.

$$x = \frac{\begin{vmatrix} -14 & 4 \\ 11 & -3 \end{vmatrix}}{\begin{vmatrix} 3 & 4 \\ -2 & -3 \end{vmatrix}} = \frac{-2}{-1} = 2, \qquad y = \frac{\begin{vmatrix} 3 & -14 \\ -2 & 11 \end{vmatrix}}{\begin{vmatrix} 3 & 4 \\ -2 & -3 \end{vmatrix}} = \frac{5}{-1} = -5$$

The solution of the simultaneous equations is then $x = 2$, $y = -5$.

Example 18 –Solve by Cramer's Rule

$$2x + 3y - z = 1$$
$$4x + y - 3z = 11$$
$$3x - 2y + 5z = 21$$

$$D = \begin{vmatrix} 2 & 3 & -1 \\ 4 & 1 & -3 \\ 3 & -2 & 5 \end{vmatrix}$$

$$D_x = \begin{vmatrix} 1 & 3 & -1 \\ 11 & 1 & -3 \\ 21 & -2 & 5 \end{vmatrix}$$

$$D_y = \begin{vmatrix} 2 & 1 & -1 \\ 4 & 11 & -3 \\ 3 & 21 & 5 \end{vmatrix}$$

$$D_z = \begin{vmatrix} 2 & 3 & 1 \\ 4 & 1 & 11 \\ 3 & -2 & 21 \end{vmatrix}$$

$$x = \frac{D_x}{D} = \frac{-312}{-78} = 4$$

$$y = \frac{D_y}{D} = \frac{156}{-78} = -2$$

$$z = \frac{D_z}{D} = \frac{-78}{-78} = 1$$

Adjoint of Matrix

- **Definition of Adjoint of a Matrix**
- **Adjoint Matrix:** Adjoint of A Matrix is the **transpose of the Cofactor Matrix**
- The adjoint of a **square matrix** $A = [a_{ij}]_{n \times n}$ is defined as the **transpose** of the matrix $[A_{ij}]_{n \times n}$, **where Aij is the cofactor of the element** a_{ij}. Adjoining of the matrix A is denoted by **adj A.**

Relation between Adjoint and Inverse of a Matrix

- To find the inverse of a matrix A, i.e A-1 we shall first define the adjoint of a matrix.
- The matrix Adj(A) is called the adjoint of matrix A. When A is invertible, then its inverse can be obtained by the formula

$$A^{-1} = \frac{adj\ A}{|A|}$$

Important Relationship

- The following relationship holds between a matrix and its inverse:
- $AA^{-1} = A^{-1}A = I$, where I is the identity matrix.

Example – Inverse of matrix

- Determine whether the matrix given below is invertible and if so, then find the invertible matrix using the above formula.

- 1 5 2, 0 --1 2, 0 0 1

Solution - A is an upper triangular matrix, the determinant of A is the product of its diagonal entries.

we have det(A) = -1

- To find the inverse using the formula, we will first determine the cofactors A_{ij} of A

$$A_{11} = \begin{vmatrix} -1 & 2 \\ 0 & 1 \end{vmatrix} = -1, \quad A_{12} = -\begin{vmatrix} 0 & 2 \\ 0 & 1 \end{vmatrix} = 0, \quad A_{13} = \begin{vmatrix} 0 & -1 \\ 0 & 0 \end{vmatrix} = 0$$

$$A_{21} = -\begin{vmatrix} 5 & 2 \\ 0 & 1 \end{vmatrix} = -5, \quad A_{22} = \begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} = 1, \quad A_{23} = -\begin{vmatrix} 1 & 5 \\ 0 & 0 \end{vmatrix} = 0$$

$$A_{31} = \begin{vmatrix} 5 & 2 \\ -1 & 2 \end{vmatrix} = 12, \quad A_{32} = -\begin{vmatrix} 1 & 2 \\ 0 & 2 \end{vmatrix} = -2, \quad A_{33} = \begin{vmatrix} 1 & 5 \\ 0 & -1 \end{vmatrix} = -1.$$

Then the adjoint matrix of A is:

(--1 --5 12, 0 1 --2, 0 0 1)

Inverse = adj A / Det A

= (1 5 -12, 0 –1 2, 0 0 1)

THANK YOU